PRACTICE MIDTERM 1 (VOJTA) - BRIEF SOLUTIONS

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(1) (a) $\frac{5}{6}$ (factor out the numerator and the denominator)

(b) 1 (factor out $(\ln(x))^2$)

(c) $-\infty = \left(\frac{\ln(3)}{0^-}\right)$

(d) 0 (calculate the left-hand-side and right-hand-side limit)

(e) Does not exist

Note: This last one is **VERY VERY VERY HARD**, and to be honest, I'm not 100 percent sure how to do this (the solution I wrote before was wrong)

(2) Notice that $0 \leq \frac{1}{x+\frac{1}{\ln(x)}} \leq \frac{1}{x}$ because $\frac{1}{\ln(x)} > 0$ when $x > 1$. Moreover, $\lim_{x\to\infty} 0 = 0$ and $\lim_{x\to\infty} \frac{1}{x} = 0$, so by the **squeeze theorem**, $\lim_{x\to\infty} \frac{1}{x+\frac{1}{\ln(x)}} = 0$

(3) (a) $7x^6 + 9x^4 - 3x^2$

(b) $\frac{-\csc(x)\cot(x)\left(x^2+x+1\right)-\csc(x)(2x+1)}{(x^2+x+1)^2}$

(c) $2xe^x \sin(x)\tan(x)+x^2e^x \sin(x)\tan(x)+x^2e^x \cos(x)\tan(x)+x^2e^x \sin(x)\sec^2(x)$

(4) (a) $[-1,1)\cup(1,3)$ (calculate the left-hand-side and the right-hand-side limits at 1 and 2)

(b) $[-1,1)\cup(1,3)$ (not differentiable at 1 because not continuous there, and differentiable at 2 because the right-hand-side and left-hand-side derivatives are equal)

(5) See review session for example (or ask me during office hours). Calculate:

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$$\lim_{x \to a} \frac{\sqrt{1+2x} - \sqrt{1+2a}}{x-a}$$

And use the conjugate form!

(6) $(y-16) = 4\left(2^3\right)(x-2)$, i.e. $y - 16 = 32(x-2)$, i.e. $\boxed{y = 32x - 48}$